Potreste darmi una mano per la soluzione di questi altri due limiti di successioni

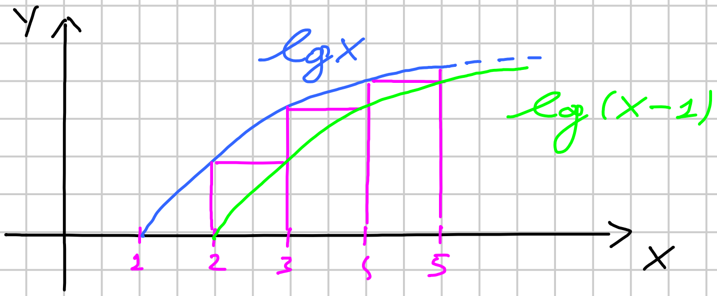
a) $\lim \dfrac{n\log n}{\log[(2n)!]}$

b) $\lim \dfrac{1+1/2+1/3+\ldots+1/n}{\log n}$

MODO 1 – CONFRONTO SERIE–INTEGRALI

a) $$\frac{n \lg n}{\lg[(2n)!]} \to \frac{1}{2}$$

$$\lg[(2n)!] = \lg(2n) + \lg(2n-1) + \ldots + \lg 1 = \sum_{k=1}^{2n} \lg k$$



$$\int_2^{2n+1} \lg(x-1)\,dx = \int_1^{2n} \lg x\,dx \le \sum_{k=1}^{2n} \lg k \le \int_1^{2n+1} \lg x\,dx$$

$$\left(\int \lg x\,dx = x\lg x - \int 1\,dx = x\lg x - x\right)$$

$$\begin{cases} \left[x\lg x - x\right]_1^{2n} \le \displaystyle\sum_{k=1}^{2n} \lg k \le \left[x\lg x - x\right]_1^{2n+1} \\ 2n\lg 2n - \cancel{2n} + \cancel{1} \le \displaystyle\sum_{k=1}^{2n} \lg k \le (2n+1)\lg(2n+1) - (\cancel{2n}+1) + \cancel{1} \\ 2n\lg 2n \le \displaystyle\sum_{k=1}^{2n} \lg k \le 2n\lg(2n+1) + \lg(2n+1) - 1 \end{cases}$$

$$\rightsquigarrow \sum_{k=1}^{2n} \lg k \sim 2n\lg 2n \qquad n \to +\infty$$

$$\frac{n\lg n}{\lg[(2n)!]} \sim \frac{\cancel{n}\lg n}{2\cancel{n}\lg 2n} = \frac{1}{2}\,\frac{\lg n}{\lg n + \lg 2} \to \frac{1}{2}$$

b) $\dfrac{1 + 1/2 + 1/3 + \dots + 1/n}{\log n} \longrightarrow 1$

$$1 + 1/2 + 1/3 + \dots + 1/n = \sum_{K=1}^{n} \frac{1}{K} \sim \log n \qquad n \to +\infty$$

MODO 2 – STIRLING

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \qquad n \to +\infty$$

a) $\dfrac{n \log n}{\log[(2n)!]} \longrightarrow \dfrac{1}{2}$

$$\log[(2n)!] \sim \log\left[\sqrt{5\pi n}\left(\frac{2n}{e}\right)^{2n}\right] =$$

$$= \frac{1}{2}\log 5\pi n + 2n \log 2n - 2n \sim 2n \log 2n$$